Note on the Electric Capacity Coefficients of Spheres.

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(Communicated by Dr. C. Chree, F.R.S. Received October 8,—Read November 21, 1912.)

In the 'Proceedings' of the Society, vol. 87, p. 109, Mr. Jeffery obtains a general solution of Laplace's equation in a form suitable for physical problems in connection with two spheres.

As an illustration he applies his solution to the problem of finding the capacity coefficients of two equal spheres, obtaining a result which he shows to be equivalent to one of Maxwell's series formulæ. He then computes a table of the numerical values of these coefficients.

Presumably in order to lessen the labour of computation, he calculates the values corresponding to equal increments of a function u, which equals $\log\{d/2r+[(d/2r)^2-1]^{\frac{1}{2}}\}$ where d is the distance between the centres of the spheres and r is the radius of either. Even in these cases, when u is small, the computation by Maxwell's formula is laborious, and this possibly accounts for the errors in the tables given.

In practical laboratory work the values of these coefficients are sometimes required, and so I gave* a table and a number of formulæ to simplify their calculation. For example, when the spheres are close together, we can use the formula

$$\frac{q_{11}}{r}=\frac{\sinh u}{2u}\left\{2{\cdot}6566572-\log 2u-\frac{u^2}{36}-\frac{49\,u^4}{21600}-\ldots\right\},$$

where q_{11} is the self-capacity coefficient of either sphere. If, for instance, we put $u = 0{\cdot}2$, we get

$$\frac{q_{11}}{r}=0{\cdot}5033400\,[3{\cdot}5729479-0{\cdot}0011111-0{\cdot}0000036-\ldots]=1{\cdot}797847\ldots.$$

The value given by Mr. Jeffery is 1·8368.

When $u = 0{\cdot}2$, $d/r = 2\cosh u = 2{\cdot}04013$.

From the tables given in my paper referred to above we find that when $d/r = 2{\cdot}04$, $q_{11}/r = 1{\cdot}79864$. It will be seen that a small change in the value of d/r makes an appreciable change in the value of the capacity coefficient.

I have recomputed the first of Mr. Jeffery's tables given on p. 119 of his paper. The values of the ratio of the diameter of either sphere to the distance between them (sech u simply) have not been accurately computed,

* 'Roy. Soc. Proc.,' A, vol. 82, p. 529.

but as these values have not been used in calculating the capacity coefficients, the errors in the latter are due to other reasons.

u ……………	0·2.	0·4.	0·6.	0·8.	1·0.	1·2.
$2r/d$ (p. 119) …………	0·9804	0·9268	0·8442	0·7526	0·6508	0·5526
$2r/d = \operatorname{sech} u$…………	0·9803	0·9250	0·8436	0·7477	0·6481	0·5523

u ……………	1·4.	1·6.	1·8.	2·0.	2·5.	3·0.
$2r/d$ (p. 119) …………	0·4654	0·3888	0·3224	0·2662	0·1630	0·0996
$2r/d = \operatorname{sech} u$…………	0·4649	0·3880	0·3218	0·2658	0·1631	0·0993

Table I.—The Self-capacity Coefficient of either Sphere.

d/r.	u.	q_{11}/r (p. 119).	q_{11}/r.
2·0401	0·2	1·8368	1·7978
2·1621	0·4	1·4796	1·4763
2·3709	0·6	1·3055	1·3073
2·6749	0·8	1·2011	1·2033
3·0862	1·0	1·1284	1·1356
3·6213	1·2	1·0916	1·0908

The remaining six values in the table are given correctly.

Mr. Jeffery also gives a table of the values of the capacity (Maxwell's definition) of a sphere in the presence of an infinite conducting plane. By utilising Kelvin's method of images and the formulæ given in my paper, we find at once that when the distance between the sphere and the plane is not greater than the radius of the sphere, we have

$$C = \frac{\sinh u}{2u}\left[2{\cdot}5407256 + \log\left(\frac{1}{u^2}\right) + \frac{u^2}{36} + \frac{7\,u^4}{21600} + \frac{31\,u^6}{1905120} + \frac{127\,u^8}{72576000} + \dots\right],$$

where C is the capacity of the sphere, or, as it is perhaps better called, the capacity between the sphere and the plane.

The nearer the sphere is to the plane the easier it is to find C by this formula. For example, when $u = 0{\cdot}2$, we find that $C = 2{\cdot}89960\ldots$, the last three terms of the formula being negligible when this accuracy only is desired. In the following table d is the distance between the centre of the sphere and the plane.

The Capacity between a Sphere and an Infinite Plane.

d/r.	u.	q_{11}/r (p. 120).	q_{11}/r.
2·0401	0·2	2·8994	2·8996
2·1621	0·4	2·2307	2·2477
2·3709	0·6	1·8982	1·8953
2·6749	0·8	1·6679	1·6679
3·0862	1·0	1·5093	1·5095
3·6213	1·2	1·3942	1·3942
4·3018	1·4	1·3083	1·3083
5·1549	1·6	1·2429	1·2430
6·2150	1·8	1·1915	1·1927
7·5224	2·0	1·1434	1·1537
12·265	2·5	1·0887	1·0887
20·135	3·0	1·0525	1·0523

The simple formulæ used for calculating the latter half of this table will be found in the 'Journal of the Institute of Electrical Engineers,' vol. 48, p. 257, and in the 'Proceedings of the Physical Society,' vol. 23, p. 352. In the 'Annalen der Physik,' vol. 27, p. 673 (1886), Kirchhoff applies Clausen's theorem to recompute some of the values given in Kelvin's well known table (Reprint 'On Electrostatics,' p. 83).

A Note on the Absorption of β-Rays.

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(Communicated by Prof. E. Rutherford, F.R.S. Received September 26,—Read November 21, 1912.)

It is clear, from the experiments of W. Wilson* and others,† that one of the main factors in the absorption of β-rays is the loss in velocity of β-rays in passing through matter. It follows from this that β-rays of a definite speed must have what may be called a maximum range, and also that an exponential law of absorption for β-rays can only be approximate.

These facts are brought out in the following experiment on the β-rays of radium E. Two preparations of radium (D + E + F) were used, one relatively weak. The weaker preparation was placed 2 cm. below an electroscope 4 cm.

* W. Wilson, 'Roy. Soc. Proc.,' 1909, A, vol. 82, p. 612; 1910, A, vol. 84, p 141.

† Crowther, 'Proc. Camb. Phil. Soc.,' 1910, vol. 15, p. 5; v. Baeyer, Hahn and Meitner, 'Phys. Zeit.,' Jan., 1911.